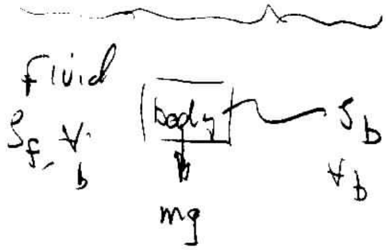
# Natural Convection






Recall
$$F_{Net} = W - F_{bouy.}$$
$$= \rho_b \forall_b g - \rho_f \forall_b g$$
$$= (\rho_b - \rho_f) g \forall$$

and volumetric expansion coef (think Thermo!)

$$\beta = \frac{1}{\forall}\left(\frac{\partial \forall}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

$$\approx -\frac{1}{\rho}\left(\frac{\Delta \rho}{\Delta T}\right)\Bigg|_{P} = -\frac{1}{\rho}\left(\frac{\rho_\infty - \rho(T)}{T_\infty - T}\right)\Bigg|_{P}$$

Solve for
$$\rho_\infty - \rho(T) = \rho(T)\,\beta\,(T - T_\infty) \quad @ \; P$$

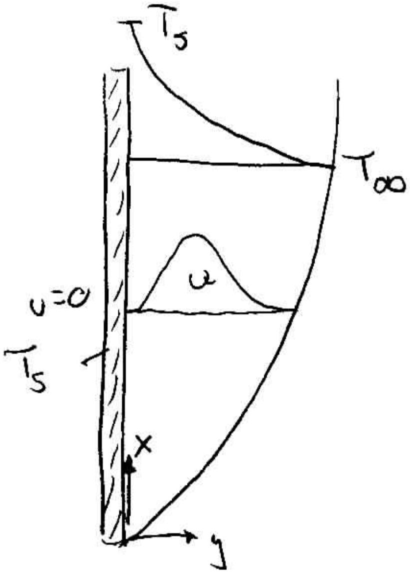

One can show for an IG
(think $P = \rho R T$)

that
$$\beta\Big|_{IG} \approx \frac{1}{T} \qquad \left[\frac{1}{T}\right]$$

Note $\beta \Delta T$ is fractional volume change for a $\Delta T$ @ P.

overall if $\Delta T = T_b - T_\infty \uparrow \Rightarrow F_{bouy.} \uparrow \Rightarrow$ Increased (natural) convection

consider natural convection on a vert. flat, hot plate...




cool fluid $u_\infty = 0, T_\infty$

This looks close to our old B.L. problem of flow along a flat plate (forced conv.) <u>except</u> we now have a buoy. force term...

$$\rho \frac{Du}{Dt} = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

Newtons law in x-momentum

$$\rho\left(\cancelto{0}{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \cancelto{0}{w}\cancelto{0}{\frac{\partial u}{\partial z}}\right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

Should be valid everywhere... like outside the B.L. ($y \to \infty$)

So
$$0 = -\frac{\partial P_\infty}{\partial x} - \rho_\infty g \quad \text{or} \quad \frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

In the B.L. $|v| << |u|$ and $\left|\frac{\partial v}{\partial x}\right| \sim \left|\frac{\partial u}{\partial y}\right| \sim 0$

Also note that Newtons Law in the y-direction tells you that $\frac{\partial P}{\partial y} = 0$.

Botton line

$$P = P(x) = P_\infty(x) \quad \text{and} \quad \frac{\partial P}{\partial x} = \frac{\partial P_\infty}{\partial x} = \underbrace{-\rho_\infty g}$$

But recall $\rho_\infty - \rho = \rho\beta(T - T_\infty)$

This initiates and drives convection. Pres. grad. ⇒ flow

Finally get

$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)}$$

Cons. of x-mom.

So putting Mass x-MOM T-energy together

(m) $$\nabla \cdot \underline{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(x-M) $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

(TE) $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

And for B.C.

At wall
$y = 0$ $\quad u = 0, \; v = 0 \quad T = T_{surf}$

Free stream
$y \to \infty$ $\quad u \to 0, \; v \to 0 \quad T \to T_\infty$

Leading edge
$x = 0$ $\quad u = 0, \; v = 0 \quad T = T_\infty$

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Non-dim. these... use charach. length $L_c$
charac. speed $V = Re_L \, \nu / L_c$
temp $(T_s - T_\infty)$

$\tilde{x} = \frac{x}{L_c}$ $\quad \tilde{y} = y/L_c$

$\tilde{u} = u/V$ $\quad \tilde{v} = v/V$

$\tilde{T} = \frac{T - T_\infty}{T_s - T_\infty}$

sub. into above eqn. to get...

for x-mom $\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \underbrace{\left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]}\frac{\tilde{T}}{Re_L^2} + \left(\frac{1}{Re_L}\right)\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}$

call this the Groshof # $= Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$

Recall $Re \sim \frac{\text{inertial forces}}{\text{viscous forces}}$

so

$Gr \sim \frac{\text{buoy. forces}}{\text{viscous forces}}$

For example for a | plate, if

laminar nat. conv. $< Gr \sim 10^9 <$ turb. nat. conv.

This Gr # shows up in mixed conv... part forced conv, part natural conv.

There is reall only one case that we can solve analytically... More about that later.

Other than that, into the lab we go...

Lots of physical configurations... correlations galore.

Lead to

$$Nu = \frac{hL_c}{k} = f(Gr_L Pr)^n = f\, Ra_L^n$$

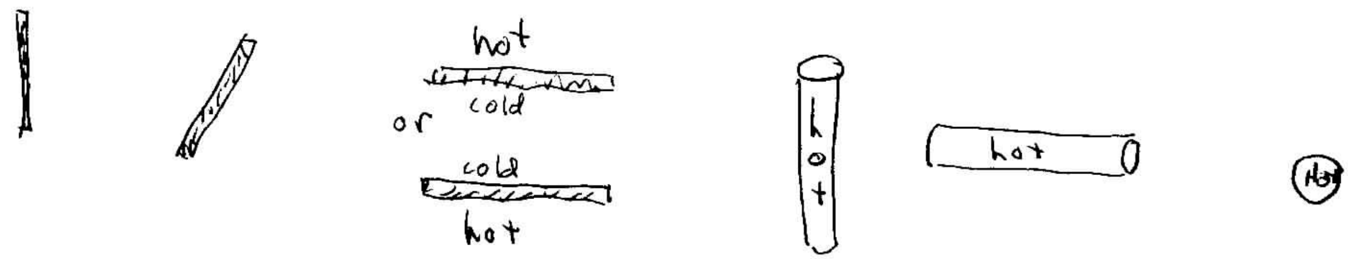
where $Ra = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha}$

<u>Typically</u> $n \sim 1/4$ laminar $n \sim 1/3$ turbulent $C < 1$

Fluid properties are evaluated at the film Temp $\boxed{T_f = \frac{T_s + T_\infty}{2}}$

$$\left.\begin{aligned} &\text{From} \quad Nu = \ldots = \text{correl.} \\ &\text{extract} \quad h = Nu \frac{k}{L_c} \\ &\text{then} \quad q_{\text{nat. conv.}} = h A_s (T_s - T_\infty) \end{aligned}\right\} \text{General procedure!}$$

Look in tables for you geometry...




Get
- $L_c$ for each
- Range of $Ra$
- $Nu$ = correlation
- $h$

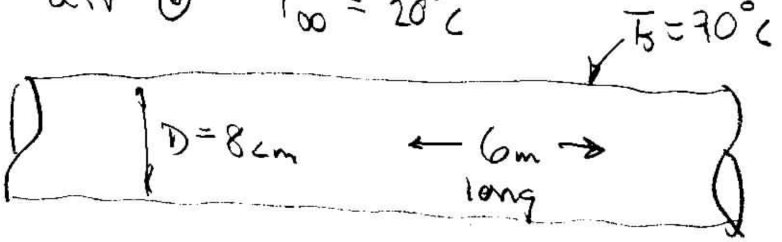
2 B.C. for ‖ plates $T_{surf} = C$ ($q''_s$ varies) or $q''_{surf} = C$ ($T_{surf}$ varies!)

Turns out $Nu$ are allmost the same if you use the $T$ half way along the plate

In that case $Nu \simeq \frac{hL}{k} = \frac{q''_{surf} L}{k(T_{L/2} - T_\infty)}$

(warning .. need iterative soln!)

Ex/ air @ $T_\infty = 20°C$




Calc $q$ by nat. conv.

Assume
- SS
- Air is IG
- $P \approx 1 atm$

Properties at film temp $T_f = \frac{T_s + T_\infty}{2} = 45°C$

for air $k = 0.02699 \frac{W}{mK}$ $\quad Pr = 0.7241$

$\nu = 1.750 \times 10^{-5}\ m^2/s$ $\quad \beta \doteq \frac{1}{T_f} = 1/318K$ (!!!)

Calc. $L_c = D = 0.08 m$ so $Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} Pr$

$= ... = 1.867 \times 10^6$

From table of correl.

$$Nu = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}} \right\}^2 = ... = 17.39$$

In which case

$h = Nu \frac{k}{D} = 5.867\ W/mK$

But $A_s = \pi D L = ... = 1.508\ m^2$

tbm

finally $q = A_s h (T_s - T_\infty) = 442 \text{ W}$ heat loss rate to air in room

But wait. What about radiation?

Gross estimate might be $q_{rad} = \varepsilon A_s \sigma (T_s^4 - T_{surr}^4)$

try $\varepsilon = 1$ overestimate but quick

$T_s$: $(70+273)$ K

$T_{surr}$: $(20+273)$ K

$q_{rad} = 553 > q_{nat.\ conv.}$

More on rad. next week.

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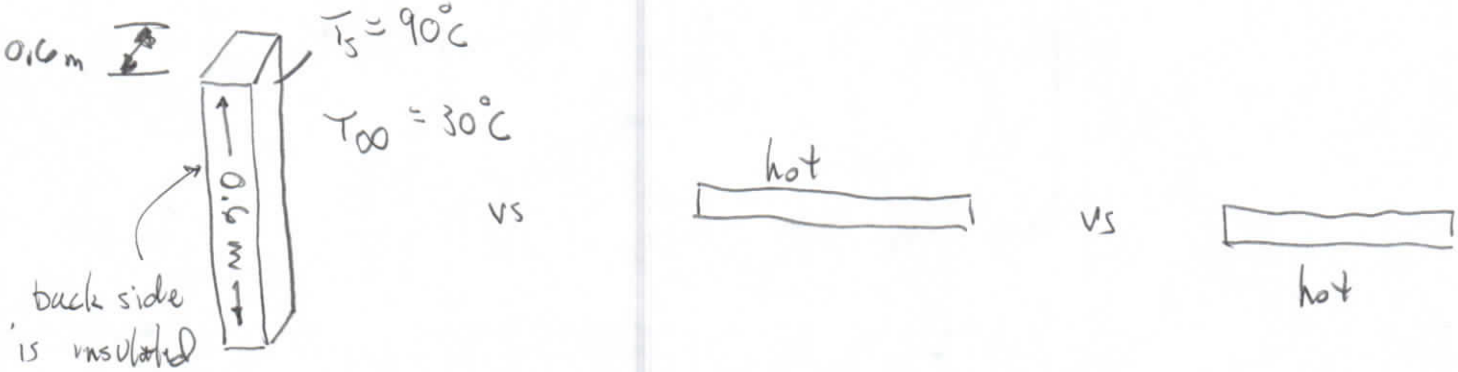
Ex| Cooling of flat plate




Calc rate of heat loss to room for each orientation

Assume
- IG
- SS
- P = 1atm

Prop. @ $T_f = \frac{T_s + T_\infty}{2} = 60°C$

$k = 0.02808 \frac{W}{mK}$ $\quad Pr = 0.7202$

$\nu = 1.896 \times 10^{-5} \frac{m^2}{s}$ $\quad \beta = \frac{1}{T_f} = \frac{1}{333K}$

Vertical $\quad L_c = L_{plate} = 0.6\,m$

so $\quad Ra_L = \dfrac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr = 7.649 \times 10^8$

Thumb through tables...

$$Nu = \left\{ 0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}} \right\}^2 = 113.3$$

(Note: a simpler $\quad Nu = 0.59\, Ra_L^{1/4} = 98.12$) 13% off

so

$h = Nu \dfrac{k}{L} = 5.302 \dfrac{W}{m^2 K} \qquad A_s = L^2 = 0.36\, m^2$

so $\quad q = h A_s (T_s - T_\infty) = 115\, W \longleftarrow$

Horizontal (hot up)

$L_c = \dfrac{A_s}{Perim} = \dfrac{L^2}{4L} = \dfrac{L}{4} = 0.15\, m$

so $\quad Ra_L = \dfrac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr = 1.195 \times 10^7$

tables $\quad Nu = 0.54\, Ra_L^{1/4} = 31.75$

$h = Nu \dfrac{k}{L_c} = 5.944 \dfrac{W}{m^2 K}$

or $\quad q = h A_s (T_s - T_\infty) = 128\, W \longleftarrow$

Horizontal (hot surf. down) $L_c$ and $\therefore$ $Ra_L$ are the same

but now $Nu = 0{,}27\, Ra_L^{1/4} = 15{,}87$

so $h = Nu \dfrac{k}{L_c} = 2.971 \dfrac{W}{m^2 K}$

or $q = h A_s (T_s - T_\infty) = 64{,}2\ W$ ⟵

Reality check for radiation...

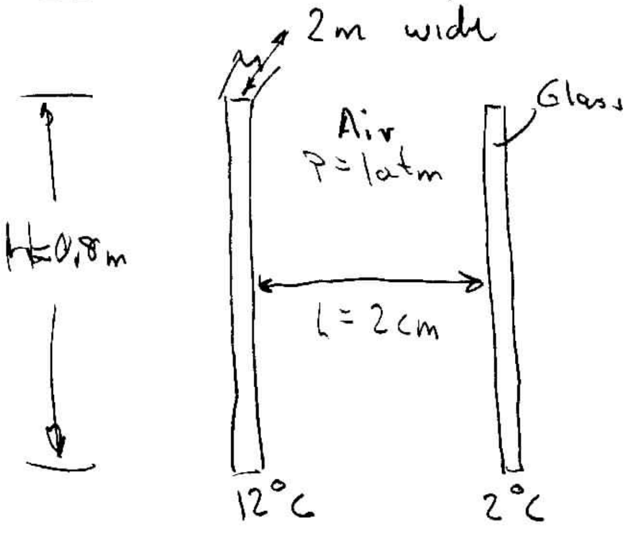
$$q_{rad} = \varepsilon A_s \sigma \left( T_s^4 - T_{surr}^4 \right)$$

$\varepsilon = 1$, $T_s = (90+273)K$, $T_{surr} = (30+273)K$

$q_{rad} = 182\ W$ ⟵

There is more to the story than simply natural convection.

Ex) ∥ Glass panes




Assume SS & IG Neglect Radiation

Prop. $\bar{T} = \dfrac{T_1 + T_2}{2} = 7°C$ $P = 1atm$

$k = 0{,}02416 \dfrac{W}{mK}$ $Pr = 0{,}7344$

$\nu = 1.4 \times 10^{-5}\ m^2/s$ $\beta = \dfrac{1}{\bar{T}} = \dfrac{1}{280K}$

Tables of correlations ... $L_c = L = 0.02\ m$ (spacing btwn plates)

$$Ra_L = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2} Pr = 1.050 \times 10^4$$

Aspect ratio $\frac{H}{L} = \frac{0.8\,m}{0.02\,m} = 40$

Tables $Nu = 0.42\, Ra_L^{1/4}\, Pr^{0.012} \left(\frac{H}{L}\right)^{-0.3} = 1.42$

Now $A_s = H \times W = 0.8\,m \times 2\,m = 1.6\ m^2$

Finally $\dot{q} = hA_s(T_1 - T_2) = \left(Nu\frac{k}{L}\right) A_s (T_1 - T_2) = 27.1\ W$

Some comments ...

Recall $Nu = \frac{hL}{k} = 1$ means (for an enclosure) pure conduction and no convection at all,

But our $Nu = 1.4$ so we infact have some natural convection occurin between the plates.

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Spherical Tank

outer cover $\varepsilon, D_o, T_o$

$D_o = 3.15\,m$

$\varepsilon = 0.9$


surrounding air $T_\infty = 20°C$

Spherical tank $D_i, T_i$

Must keep $T_o \le 45°C$

Is the air gap sufficient?

Assume
- SS
- IG
- $P = 1\,atm$
- $T_{surf} = \text{const}$
- Radiation btwn walls is neg.
- $T_{surr} = T_\infty$

Prop

$\bar{T}_{avg} = \frac{T_i + T_o}{2} = 95°C$ + $P = 1\,atm$ gives

$k = 0.03060\ \frac{W}{mK}$ $\quad Pr = 0.7122$

$\nu = 2.254 \times 10^{-5}\ \frac{m^2}{s}$ $\quad \beta = \frac{1}{\bar{T}} = \frac{1}{368\,K}$

Air Inside tank and outer shell

$T_{film} = \frac{T_o + T_\infty}{2} = 30°C$ + $P = 1\,atm$

$k = 0.02588\ \frac{W}{mK}$ $\quad Pr = 0.7282$ $\quad \varepsilon = 0.9$

$\nu = 1.608 \times 10^{-5}\ \frac{m^2}{s}$ $\quad \beta = \frac{1}{T_f} = \frac{1}{303\,K}$

outside outer shell

For natural convection between concentric spheres

$$T_{avg} = 95\,^\circ C \quad \text{(assumed)}$$

$$\frac{T_o + T_i}{2} = T_{avg} \Rightarrow T_o = 2T_{avg} - T_i = 40\,^\circ C$$

so
$$Ra_L = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2} Pr = 1{,}7342 \times 10^6$$

where
$$L_c = \frac{D_o - D_i}{2} = 0{,}075 \text{ m}$$

The effective thermal conductivity is

$$F_{sph} = \frac{L_c}{(D_i D_o)^4 \left(D_i^{-7/5} + D_o^{-7/5}\right)^5} = 0{,}0007602$$

$$k_{eff} = 0{,}74\, k \left(\frac{Pr}{0{,}861 + Pr}\right)^{1/4} \left(F_{sph} Ra_L\right)^{1/4} = \ldots = 0{,}1119 \ \frac{W}{mK}$$

For the natural convection from the outer shell to surr. air

$$Ra_D = \frac{g\beta(T_o - T_\infty)D_o^3}{\nu^2} Pr = 5{,}6999 \times 10^{10}$$

Now

$$Nu = 2 + \frac{0{,}589\, Ra_D^{1/4}}{\left[1 + \left(\frac{0{,}469}{Pr}\right)^{9/16}\right]^{4/9}} = 224{,}69$$

so that

$$h = Nu \frac{k}{D_o} = \ldots = 1{,}846 \ W/m^2K$$

Finally $\quad q_{enclosure} = q_{conv} + q_{rad}$

absolute $T_s$!

$$K_{eff}\,\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o) = h(\pi D_o^2)(T_o - T_\infty) + \varepsilon\sigma(\pi D_o^2)\left(T_o^4 - T_{surr}^4\right)$$

leads to $T_o = 40{,}6\,^\circ C$ (numerical root)

So no law suits for burned fingers!

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